

## *Paradox*

In normal thought, if you follow a line of reasoning and arrive at a contradiction (a proposition that is both true and false), then you have gone wrong somewhere. The solution may be obvious (if a word in the conclusion is ambiguous), or you can trace the steps in your reasoning for an error. If that is all fine, then your initial assumptions were wrong. Paradoxes arise when you can't see anything wrong with your assumptions or your reasoning, and yet you still arrive at a contradiction. A drastic response would be to say you have thus proved that reality contains contradictions. The normal response is to assume a deep error in the reasoning, which needs more research.

Assuming the world does not contain contradictions, then all paradoxes are failures in thinking. There is no precise categorisation of paradoxes, but some concern the world (involving space, time and time travel, or movement), others seem to result from ordinary language, others come from very formal systems, and others involve our decision-making. On the whole paradoxes concern very precise contradictions. There are also bigger conflicts in thought which are provocative **puzzles** (Greek *aporiai*) where it is hard to choose between two explanations, or **antinomies**, where there seem to be only two explanations of some major issue, and neither of them seems acceptable.

The most interesting paradoxes are those that just won't go away. The **Monty Hall** paradox (concerning the probability of a quiz show prize being in a certain box) is an example in which popular thought is contradicted by accurate thought, so it reveals our weakness, but not a deep truth. The so-called **Barber** paradox (which asks who shaves a barber who follows the rule 'only shave those who don't shave themselves') proved that in set theory you can't assume that every concept generates a set. Consequently modern set theory relies on axioms instead of concepts, and the Barber is a historical footnote. There is also a paradox concerning the greatest ordinal number, which has led to revisions in the set theory account of large ordinal numbers. These paradoxes are permanent dead-end signs for certain lines of thought.

More interesting are paradoxes which still grip us, even though they have been overtaken by modern modes of thinking. The **Achilles** paradox is a nice example. If a slow tortoise is allowed to start ahead of the great athlete Achilles in a race between them, then to win Achilles must always first reach the place where the tortoise is now located, but the tortoise is moving, so it won't be there when Achilles arrives. Achilles repeats the chase to the tortoise's second location, but it has again moved on. It seems that this repetition can never be broken, even though the gaps get progressively smaller. So a fast runner can't overtake a slow runner! The problem involves the division of space and time into endlessly minimised 'infinitesimals', but mathematicians have now abandoned those. There is no neat solution to the paradox, though, and it seems to reveal some failure in our ordinary thought about space and time. For example, if the present moment is just where past meets future, then nothing can happen in the present!

Puzzling errors in our own thinking are revealed when I believe there is no way my ticket will win a huge **lottery**, and yet I'm sure someone will win it, or the **preface** to my book apologises because the book certainly contains errors, even though I've checked the book and don't actually believe there are any. Vague talk can seem fine, until I carefully check its implications, as when I say there will be a **surprise test** this week, but then see it can't be on the last day (because that wouldn't be a surprise), or on the penultimate day (because that therefore couldn't be a surprise either), and so on. It seems that no teacher should promise to give a surprise test!

Dozens of paradoxes have been discovered, and there will probably be many more, but there are two which have enduring interest for philosophers, one concerning how we speak, and the other concerning how we understand the world. The ancient **Liar Paradox** is the simplest and most dramatic of paradoxes, because it just requires someone to say "This sentence is not true". An attentive listener must decide whether they agree, but if they do agree then the sentence says they shouldn't have agreed, but disagreement implies that the sentence is true, and so they should have agreed with it. We might forbid anyone to speak such a sentence, or claim that the sentence is meaningless. If you suggest that it is neither true nor false, you will be invited to contemplate the sentence 'this sentence is neither-true-nor-false or it is not true' (which is a 'Strengthened Liar'). Most discussion, however, has pointed towards new theories about the nature and role of the word 'true'. The problem seems to come from 'true' being part of the language being used, when it might be better understood when we step outside the language (into a 'meta-language'), and say in the metalanguage 'the sentence "p" is not true in the language we are discussing (the 'object-language')'. This led to a whole new definition of truth (the 'semantic' theory). However, the problem won't go away, so this paradox is right at the centre of modern studies of the concept of truth.

The other big problem, concerning our grasp of the world, is the paradox of the **Heap** (the **Sorites Paradox**). One stone is not a heap, and two stones are not a heap, but a hundred piled up stones are a heap. Some individual stone must have turned it into a heap, but we can't say which one. If red shades off into orange, we can't seem to say which is the first orange shade. The paradox expresses the problem of vagueness, as seen in small increments (such as loss of hairs resulting in baldness). One response is to say that the fourth stone begins the heap, because three others can raise it up (the that's not much of a 'heap'), another response is to say that it is not true that one particular stone triggers a heap, and a drastic response denies that heaps exist at all. A related puzzle concerns when stones are replaced, rather than added, since the resulting heap is and isn't the same as the original.

It might appear that paradoxes are of interest to minds that love logical precision, but should be ignored by wiser people who are not side-tracked by intriguing details. The paradox-lovers are likely to reply, though, that the 'wiser' people are rather superficial in their thinking, because if you follow the trail of these oddities, they reveal that whole systems of major thought are resting on very wobbly foundations. If the Liar really says that we don't understand the word 'true', or the Heap really says that we don't understand some of our own concepts, or the Achilles says that we don't really understand such an obvious phenomenon as movement, then philosophers really have to take paradox seriously. For logicians, of course, paradox is the main reason why they can't sleep at night.